Elastic and Inelastic Scattering at 168 MeV in the $O^{16}-C^{12}$ System*

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(Received 30 January 1964; revised manuscript received 31 March 1964)

Differential cross sections for the elastic scattering and the inelastic scattering to the most strongly excited states in the $O^{16}-C^{12}$ system have been measured using the 168-MeV O^{16} beam from the Yale University heavy-ion linear accelerator. The states most strongly excited are the 2^+ state in C^{12} at 4.43 MeV and the 3^- state in O^{16} at 6.14 MeV. The mutual excitation reaction wherein both these levels are excited ($Q=-10.57$ MeV) was also observed and measured. All of the angular distributions measured have an oscillatory behavior, a consequence of the strong absorption of the incident wave in the nuclear interior and indicative of a direct interaction mechanism. A distorted-wave Born approximation analysis of the data is obtained in order to extract reliable nuclear parameters. The results of this experiment are compared to previous elastic and inelastic scattering, from C^{12} in the adiabatic-Fraunhofer limit. Fair agreement in shown in spite of the variety of projectiles used in the comparison. The transition strengths for inelastic scattering are further compared to the electromagnetic transition probabilities and a scheme for relating the two quantities is presented.

I. INTRODUCTION

THE elastic and inelastic scattering of spin 0, 10.5-MeV/amu heavy ions can be understood in much the same framework that describes the scattering of medium-energy (25–50 MeV) alpha particles. The observed angular distributions show pronounced oscillatory structure with a periodicity dependent on the momentum transferred in the collision process; and further, the states most strongly excited in these reactions are those having an enhanced electric transition probability to the ground state. Both of these features are reproduced theoretically by assuming a strongly absorbing, deformed nucleus as in the adiabatic-Fraunhofer theory[1] or in the distorted-wave Born approximation (DWBA) method.[2–4] In fact, in spite of great differences in projectile type, the inelastic scattering of high energy electrons and the inelastic scattering of nuclear projectiles ranging from protons to heavy ions of sufficiently high energy that compound-nucleus contributions to the yield of the observed level are negligible, show nearly the same relative excitation of these strongly excited states. This similarity can be understood if the interaction between the target and the projectile is expanded in a multipole series.[2,5,6] In this expansion, for small momentum transfer, the spin-independent part of the transition strength for inelastic scattering is the same for the various projectiles and has the same form as the electric multipole operator connecting the ground and excited states.[7]

Recent work of Garvey *et al.*[8,9] showed two ways in which heavy-ion reactions differ from the inelastic scattering of lighter projectiles. In the scattering of 125-MeV C^{12} from C^{12} they observed and obtained the angular distribution for the reaction in which both C^{12} nuclei are excited to their first excited state ($Q=-8.86$ MeV). Such reactions are of course not possible with the lighter projectiles as they have no bound excited states. This process in the C^{12}–C^{12} system has a large cross section, averaging 5 mb over the range of angles observed, and is larger than the elastic cross section beyond 30° in the center of mass. The angular distribution was fitted reasonably well employing a Born approximation calculation, with two-body interactions. Although this scattering process is conceptually simple it is difficult to incorporate into the DWBA programs in use.

In addition to the mutual excitation reaction, a sizable cross section has been measured[10] for the excitation of a state at 14.0±0.5 MeV in C^{12}, which has subsequently been shown[9] to be most probably a 4^+ state belonging to the ground-state shell-model configuration. This state has not shown up strongly in other (x,x') scattering experiments; presumably because they involved smaller amounts of momentum transfer or the excitation of this state was not seen because of alpha-particle background

* This work was performed under the auspices of the U. S. Atomic Energy Commission and is based in part on a dissertation submitted by J. C. Hiebert in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Yale University. A preliminary report of this work was presented at the Third Conference on Reactions Between Complex Nuclei, Asilomar, California, April 1963.

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[1] J. S. Blair, Phys. Rev. **115**, 928 (1959); S. I. Drozdov, Zh. Eksperim. i Teor Fiz. **28**, 736 and 734 (1955) [English transl.: Soviet Phys.—JETP **1**, 588 and 591 (1955)]; E. V. Inopin, *ibid.* **31**, 901 (1956) [English transl.: *ibid.* **4**, 764 (1957)].

[2] E. Rost and N. Austern, Phys. Rev. **120**, 1375 (1960).

[3] R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).

[4] E. Rost, Phys. Rev. **128**, 2708 (1962).

[5] R. H. Lemmer, A. de Shalit, and N. S. Wall, Phys. Rev. **124**, 1155 (1961).

[6] N. Austern, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960), p. 323.

[7] W. T. Pinkston and G. R. Satchler, in *Proceedings of the International Conference on Nuclear Structure, Kingston 1960*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960), p. 394.

[8] G. T. Garvey, A. M. Smith, J. C. Hiebert, and F. E. Steigert, Phys. Rev. Letters **8**, 25 (1962).

[9] G. T. Garvey, A. M. Smith, and J. C. Hiebert, Phys. Rev. **130**, 2397 (1963).

[10] K. H. Wang, S. D. Baker, and J. A. McIntyre, Phys. Rev. **127**, 187 (1962).

problems.[11] Thus heavy ions may be quite useful in uncovering high-lying $T=0$ collective states that are not strongly excited in interactions with light projectiles.

This paper reports a study of the scattering of 168-MeV O^{16} from C^{12} and relates the results to previous experiments. Preliminary work on this system has been reported by Williams and Steigert.[12] Their experimental energy resolution was typically 2 MeV, full width at half-maximum (FWHM), and they were unable to resolve any of the O^{16} states. The present experiment, with improved energy and angular resolution and employing kinematic coincidence techniques, has been able to resolve some of the O^{16} excitations and detect another example of a mutual excitation reaction.

II. EXPERIMENTAL

(a) Apparatus

Figure 1 shows the essential features of the scattering chamber which is described in detail in Ref. 9. The magnetically analyzed 168-MeV O^{16} beam from the Yale heavy-ion accelerator was passed through a pair of collimating slits and scattered from an evaporated C^{12} target. Two independently movable semiconductor detectors were positioned in a plane containing the beam and could be operated in fast coincidence for kinematic separation of reactions. The detectors could not be placed forward of 6° in the laboratory because all beam monitoring was accomplished with the installed Faraday cup.

The rapid oscillation with laboratory angle of the observed yield for the reactions under study is indicated in Fig. 2. The successive maxima are moved closer together in the laboratory by the center-of-mass transformation in this reaction than would be the case for the C^{12} on O^{16} reaction. The latter reaction also offers better absolute energy resolution because of the lower energy of the C^{12} beam from the accelerator. However, the need for high purity targets of several hundred $\mu g/cm^2$ thickness made a C^{12} target the obvious choice. Brief experiments with a thin ice target (~ 1 mg/cm^2), did give encouraging results but were not continued. Thus, energy resolution was achieved by requiring sharp angular resolution. The angular definition of the ap-

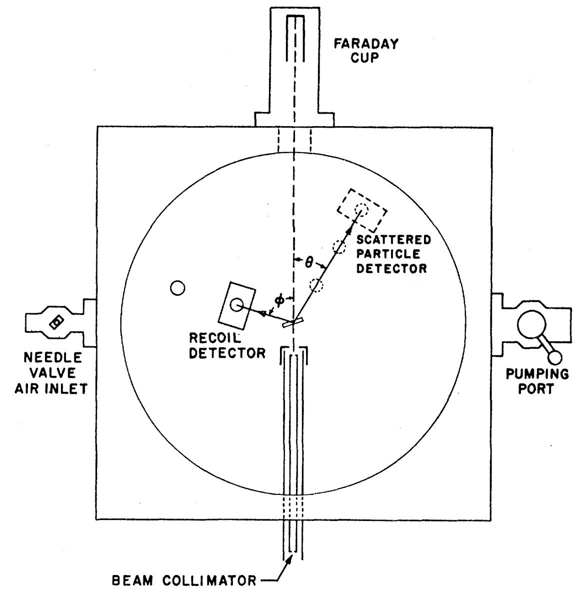

FIG. 1. Schematic diagram of the scattering chamber.

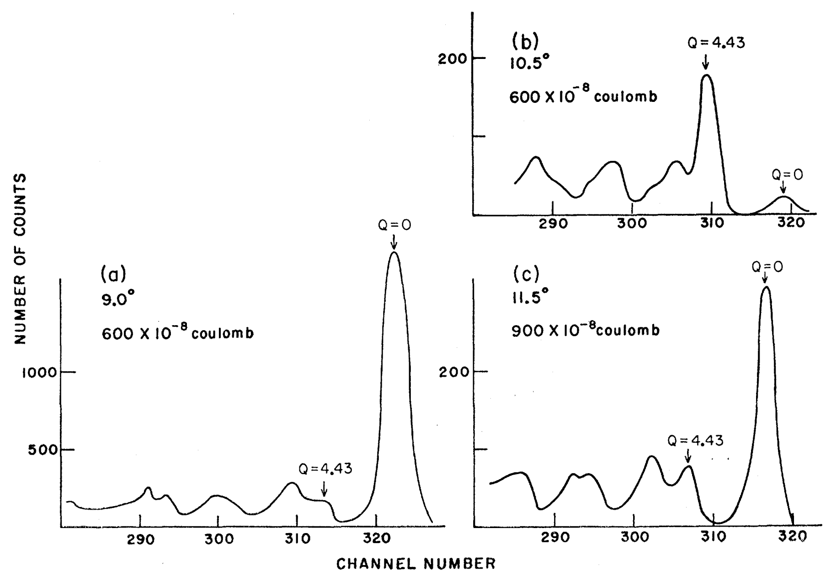

FIG. 2. Sample energy spectra of the scattered particles at three laboratory angles illustrating the rapid variation of the cross sections with angle.

[11] University of Washington, Cyclotron Progress Report, 1963 (unpublished).
[12] D. J. Williams and F. E. Steigert, Nucl. Phys. 30, 373 (1962).

paratus for this experiment was determined by a pair of rectangular slits 0.049×0.098 in. located 12 in. apart in the beam collimator together with vertical slits 0.049 in. wide in front of both detectors. This fixed the range of scattering angles seen by the scattered particle detector at $\Delta\theta = 50'$ for $\theta = 14°$. To determine absolute zero for the scattering angle, small-angle Rutherford scattering from gold was measured for both positive and negative angles. This check was made periodically during a three-day run and the beam drift with respect to the chamber axis was found to be less than $\pm 3'$.

The electronics used was developed at Yale by Gingell.[13] This completely transistorized system included double delay-line amplifiers, single-channel analyzers, a biased post amplifier[14] and slow-fast coincidence units.[15] Two RIDL-400 channel analyzers were used and gated appropriately to store the desired information.

The energy spread of the O^{16} beam, measured by observing the elastic scattering from a 170 $\mu g/cm^2$ evaporated gold target was about 500 keV full width at half-maximum (FWHM). The stability of the scattered-particle energy-analysis system was better than 1% over a three-day run.

(b) Procedure

Figure 3 shows the presently known[16] low-lying energy levels of C^{12} and O^{16}. Two experimental techniques were used to separate inelastic-scattering reactions. The first used a biased post amplifier to insure that the energy resolution of the scattered-particle detector was fully displayed in a 400-channel analyzer. The second technique was the use of kinematic coincidence. This technique was necessary for separating the mutual excitation reaction in which the 4.43-MeV state in C^{12} and the 6.14-MeV state in O^{16} are excited from other reactions with a Q value near -10.57 MeV. In some cases the angular resolution of the detectors was inadequate for kinematic separation of the reactions. However, in all these cases the state excited is unstable to particle emission and thus should not produce a kinematic coincidence. Kinematic coincidence was also used to make checks on the separation of reactions as accomplished with the biased post amplifier.

It should be noted that the energy spectra of the scattered projectiles will only exhibit groups corresponding to reactions leaving the projectile in bound states. Thus the excitation of a state in O^{16} above 7.16 MeV will not be observed in the scattered-particle spectra unless the state has a large branching ratio for gamma decay.

[13] C. E. L. Gingell, IRE Trans. Nucl. Sci. **NS-10**, No. 3 (1963).

[14] T. L. Emmer, IRE Trans. Nucl. Sci. **NS-9**, No. 3 (1962).

[15] R. L. Chase, Rev. Sci. Instr. **31**, 945 (1960).

[16] F. Ajzenberg-Selove and T. Lauritson, Nucl. Phys. **11**, 1 (1959); *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C., 1962), Sets 5 and 6.

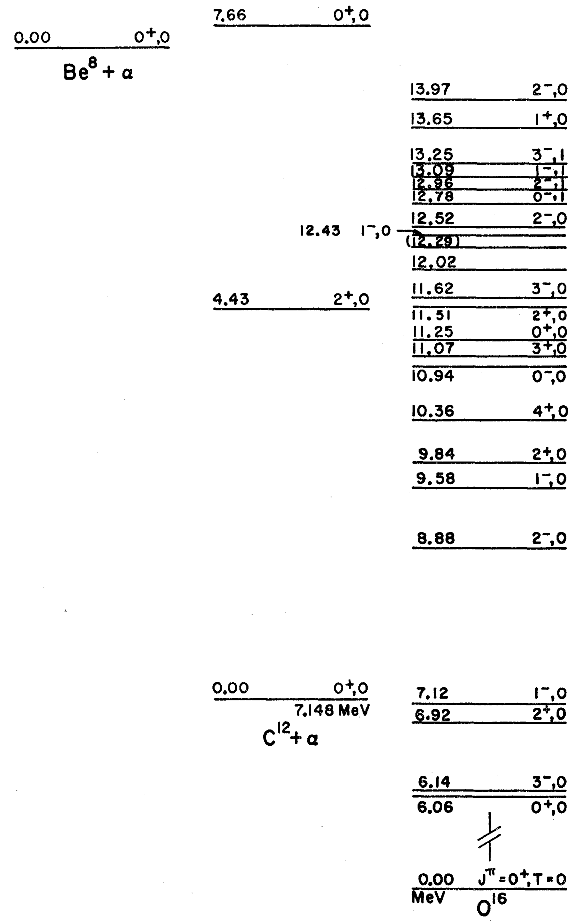

FIG. 3. Energy levels in O^{16} as taken from Ref. 16.

All natural parity states, $\pi = (-)^L$ where L is the angular momentum of the state, in O^{16} above 7.16-MeV decay principally through alpha-particle emission to C^{12}.

Although the unnatural parity states of O^{16} cannot decay to the C^{12} ground state, it can be shown that in the scattering of spinless projectiles from even-even nuclei the excitation of unnatural parity states is forbidden to first order. Experiments with alpha particles[17] have shown that the strength of these excitations decreases as the incident energy increases. This would indicate that unnatural parity states should be weakly excited in the reaction under study. This was found to be true for a particular level by looking with some care for the excitation of the 2^- state in O^{16} at 8.88 MeV.

Figure 4 shows an energy spectrum of the scattered particles at 8.25° laboratory angle obtained with a 130 $\mu g/cm^2$ evaporated C^{12} target. Figure 4(A) shows the scattered-particle energy spectrum displayed in a 400-channel analyzer. The arrows indicate the expected positions of peaks corresponding to the reactions of

[17] W. W. Eidson and J. G. Cramer, Jr., Phys. Rev. Letters **9**, 497 (1962); J. G. Cramer, Jr. and W. W. Eidson, Bull, Am. Phys. Soc. **8**, 26 (1963).

interest. Figure 4(B) shows this same spectrum as displayed in a second 400-channel analyzer after being expanded by a factor of two with a biased post amplifier. In this spectrum there is a peak at about 7 MeV resolved from the 6.14-MeV peak. It is not entirely clear whether the peak at 7 MeV is due to the excitation of one or both members of the doublet in O^{16} at 6.92 and 7.12 MeV. This inadequacy in energy resolution also applies to the O^{16} doublet at 6.06 and 6.14 MeV. Previous work[9,10] with the C^{12}–C^{12} system has shown the 0^+ state in C^{12} at 7.66 MeV to be very weakly excited. The results of this experiment indicate that again, the 7.66-MeV state in C^{12} is weakly excited and also that the 0^+ state in O^{16} only weakly contributes to the observed peak near 6 MeV.

Another separation of peaks is suggested in channels 60–67 of Fig. 4(B). The peak in channel 62 is thought to be due to the neutron transfer reaction $C^{12}(O^{16},O^{17})C^{11}$ with $Q=-14.75$ MeV, but a positive kinematic identification of this peak was not made. The peak at the expected position for a reaction with $Q=-14.05$ MeV corresponds to the excitation of a 4^+ state in C^{12} at approximately 14 MeV.[9]

For all events with a Q value greater than 5 MeV there is ambiguity in the assignments to reactions because of the density of states in these nuclei. Although the energy spread of the beam of less than 0.5 MeV (FWHM) could be achieved by restricting the angular resolution, a compromise between energy resolution and intensity was necessary because of the limited accelerator time available. Thus, the data for this experiment were taken with a system energy resolution of 0.9 MeV (FWHM).

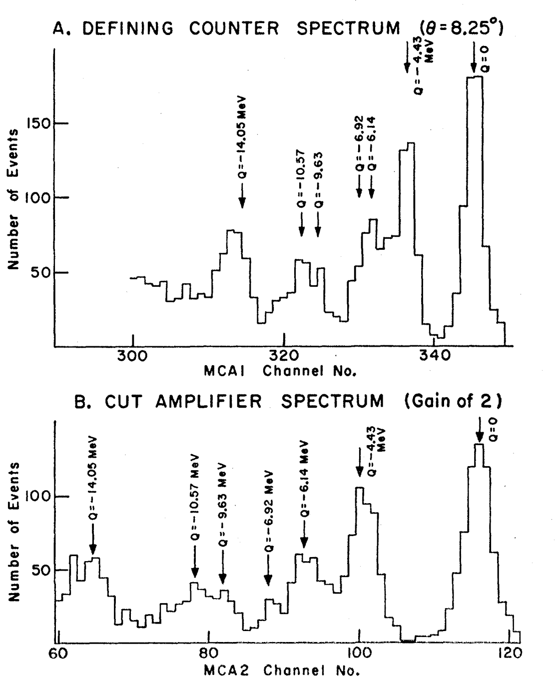

FIG. 4. Energy spectrum at 8.25° in the laboratory of 168-MeV O^{16} nuclei scattered from C^{12}. (B) shows the same spectrum as (A) expanded by a factor of 2 with a biased amplifier.

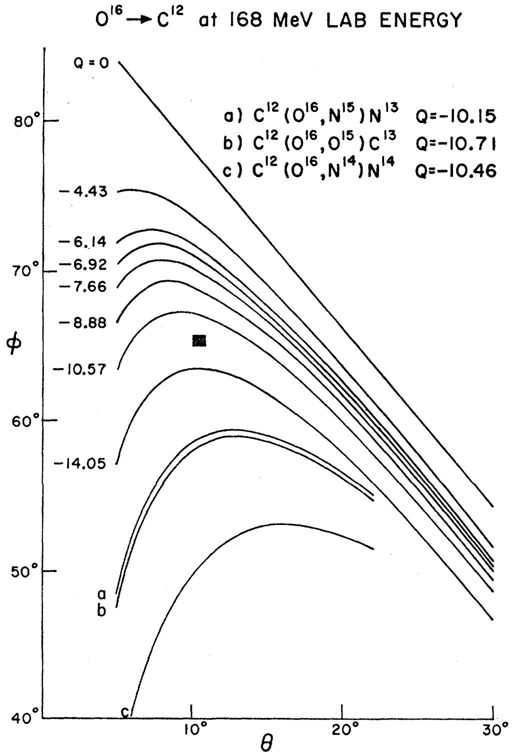

FIG. 5. Graph of the kinematic relationship between scattering angle θ and recoil angle φ for typical O^{16}–C^{12} reactions.

The kinematic separation of reactions is illustrated in Fig. 5 where the relation between scattering angle θ and recoil angle φ of the target nuclei is plotted for several of the reactions of interest, as identified by their Q value. The plots are obtained using a high-speed computer solution of the relativistic Q equation. The solid block in the figure indicates the maximum range of angles seen by the detectors, determined for a typical scattering angle. For scattering angles less than 20° the angular separation between reactions becomes appreciable although it is still impossible to separate contributions to the individual states in the O^{16} doublets mentioned above.

Figure 6 illustrates how the scattered-particle energy spectra vary with recoil angle when the 400-channel analyzer is gated on by fast coincidence pulses. These spectra were taken using a 445 $\mu g/cm^2$ evaporated C^{12} target that was used throughout the experiment. The statistics are poor in these spectra which are shown only for illustrative purposes. The spectrum at the top of the figure is an energy-gated analyzer spectrum of the scattered particles at 13.9° within approximately 20 MeV of the elastic peak. The arrows, reading from the right, refer to the expected location of peaks corresponding to reactions with Q values of 0, -4.43,

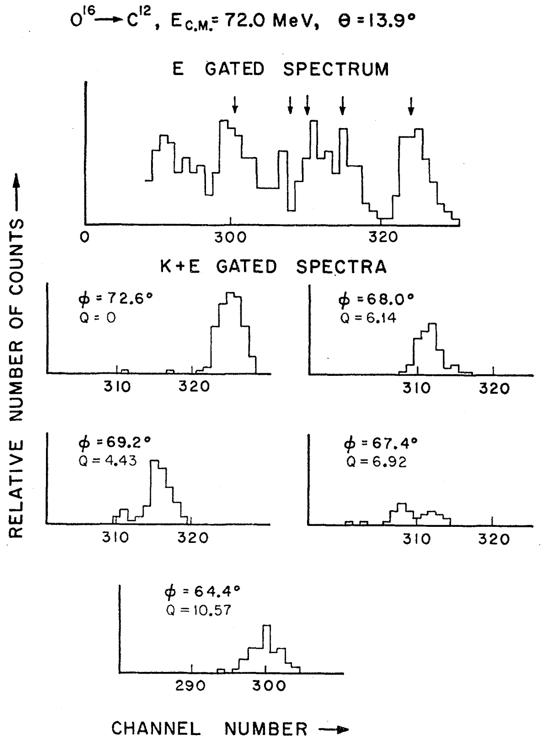

FIG. 6. Typical spectra showing separation of reactions with kinematic coincidence. Scattered-particle spectra are observed at 13.9° in the laboratory for various recoil angles.

-6.14, -6.92, and -10.57 MeV, the only large two-body cross sections observed. The remaining spectra in the figure are the kinematic-gated analyzer spectra of the scattered particles when the recoil angle is set at the optimum angle for each of these reactions.

The Q value of the mutual excitation reaction in which the 4.43-MeV state in C^{12} and the 6.14-MeV state in O^{16} are excited is -10.57 MeV. The large number of states in both these nuclei near this reaction Q poses a serious resolution problem. Consider, for example, the 4^+ state in O^{16} at 10.36 MeV. This state cannot be resolved from the mutual excitation either through energy or angular resolution. It is possible to resolve the two reactions only because the 10.36-MeV state in O^{16} is unbound with respect to alpha-particle emission and decays to the ground state of C^{12} in about 10^{-20} sec. A scattered O^{16} nucleus reaches the detector in about 10^{-9} sec so the decay occurs effectively at the target. Two considerations combine to produce the result that the excitation of the 10.36-MeV state will not contribute counts in the scattered-particle spectra at the energy expected for a reaction with $Q=-10.36$ MeV. First, the depletion depth of the scattered-particle detector was adjusted to correspond to the range of 168 MeV O^{16} nuclei. When the excited O^{16} nucleus breaks up into $C^{12}+\alpha$, either both the decay products or just the alpha particle pass through the sensitive region of the detector, and part of the energy is not deposited in the detector. Thus, in the improbable event where both the decay products enter the detector, the energy deposited in the detector does not correspond to the energy deposited by stable O^{16} nuclei from a reaction with $Q=-10.36$ MeV. Second, if either or both the decay products are not detected in the scattered-particle detector the event will be recorded at a lower energy than expected for O^{16} nuclei, or not at all. The precise angular distribution of the decay products is unknown due to the uncertainty in population of the magnetic substates of the excited O^{16} nuclei. However, as discussed in detail elsewhere,[9,18] conservative estimates show that the probability of detecting both decay products in the scattered-particle detector is less than 1 in 10^4 scattering events in which the scattered particle decays by particle emission. Because of these two considerations and the fact that unnatural parity states are weakly excited it is assumed that there is no contribution to the kinematic-gated spectra, when the detectors are set to observe the mutual excitation reaction, from any states in O^{16}.

There are also reactions exciting states in C^{12} which might be indistinguishable from the mutual excitation reaction being studied. The only excited state in the region of interest which has been observed in previous heavy ion scattering[9] is the 3^- state at 9.63 MeV. Smith has made a detailed estimate[18] of the contribution of this state to the kinematic coincidence counting rate under optimum conditions and has shown it to be less than 3% of the noncoincidence yield from this state. Experimental checks in both the C^{12}–C^{12} and O^{16}–C^{12} systems show this estimate to be an upper limit.

The experimental procedure for determining the angular distribution of the mutual excitation reaction from data obtained using kinematic coincidence techniques has been described in Ref. 9. Briefly, the efficiency for the detection of two-body reactions is determined at each angle from the elastic-scattering yield. With the recoil detector positioned for the $Q=0$ reaction an ungated pulse-height spectrum of the scattered-particle detector is stored in one analyzer while, simultaneously in another analyzer, only pulses from the scattered-particle detector that have an associated kinematic coincidence are stored. Then the recoil detector is moved to the correct angle for the $Q=-10.57$ MeV reaction and the pulses in kinematic coincidence are again stored. The experimental yield for the mutual excitation reaction is given, with suitable corrections, by the ratio

$$N_{\text{gated}}(10.75)\times\frac{N_{\text{ungated}}(0)}{N_{\text{gated}}(0)},$$

where $N_{\text{gated}}(10.57)$ refers to the number of counts in the 10.57-MeV peak in the gated analyzer, etc.

[18] A. M. Smith, doctoral dissertation, Yale University, 1962 (unpublished).

(c) Absolute Cross Sections

The absolute differential cross section for the elastic scattering of O^{16} from C^{12} was determined at a laboratory angle of 8° 57′ by comparing it to the elastic scattering from a 170 $\mu g/cm^2$ evaporated gold target at the same angle. The O^{16}–Au^{197} scattering is assumed to be described by the Rutherford formula and measurements indicated that this is indeed the case. This absolute cross-section measurement is believed to be accurate to $\pm 15\%$. The data for this experiment were collected on three separate runs and the absolute cross-section calibrations made on each of these occasions were consistent to $\pm 6\%$.

III. RESULTS

A composite of the measured cross sections is shown in Fig. 7. The cross section for the excitation of the state in C^{12} near 14 MeV was not measured because of the large uncertainties in the background contributions. A rough angular distribution of the 14-MeV peak indicated that the slope of the cross section was typical of a 2nd-order excitation at backward angles but forward of $\Theta_{c.m.} = 30°$ another reaction seems to be contributing, probably the neutron-transfer reaction mentioned above.

The $Q = -7.0 \pm 0.1$-MeV cross section is attributed principally to the excitation of the 2^+ state in O^{16} at 6.92 MeV, but there may well be a contribution from the 1^- state at 7.12 MeV. The cross sections shown have relative errors on the order of $\pm 10\%$ at the maxima of the oscillations and the order of $\pm 20\%$ in the minima. A high-resolution experiment is being attempted by Newman and collaborators at Oak Ridge National Laboratory with a 42-MeV O^{16} beam to determine whether or not the 1^- state is excited in heavy-ion collisions.[19]

A geometric correction has been applied to the elastic-scattering data. This correction is important because the angular acceptance of the scattered-particle detector is significant compared to the very sharp oscillations in the elastic yield. The effect of the finite acceptance angle is to reduce the maxima of the oscillations on the yield and particularly, to fill in the valleys. The calculation of this correction follows the method of Silverstein.[20]

The cross sections in Fig. 7 illustrate the operation of the Blair phase rule[1] for states in both the target and the projectile, a generalization of the original rule derived only for states in the target. The cross sections for the 2^+ excitations in O^{16} and C^{12} are out of phase with the elastic scattering. This indicates the equal status of target and projectile with regard to the Blair phase rule. The mutual-excitation cross section decreases less steeply with scattering angle than do the first-order excitations. This effect was also observed for the mutual excitation in the C^{12}–C^{12} system[9] and is predicted by a plane-wave Born approximation calculation.

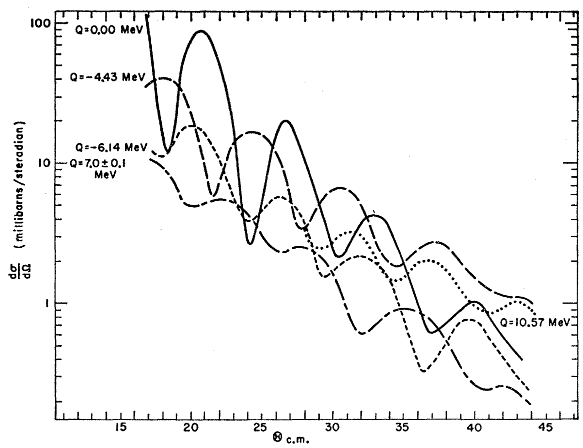

FIG. 7. Measured differential cross sections versus center-of-mass angle for the scattering of 168-MeV O^{16} ions from C^{12}.

An upper limit of 0.36 mb/sr for the excitation of the lowest unnatural parity state in O^{16}, a 2^- state at 8.88 MeV, was determined at $\Theta_{c.m.} = 27.8°$. This limit was set by positioning the recoil detector at the appropriate angle for detecting this reaction and observing in the kinematic coincidence gated spectrum of the scattered particle detector the number of counts in the proper energy interval. The number of counts in this interval was not significantly larger than the background counting rate and the cross section for the excitation of this state is estimated to be at least a factor of 20 times weaker than the excitation of the 6.14-MeV state in O^{16}.

(a) Distorted-Wave Born Approximation Analysis

The distorted-wave Born approximation (DWBA) assumes a deformed potential-well interaction based on the collective model. The relative motion in the collision is described by distorted waves, which include the elastic scattering and are calculated using an optical-model potential. The Oak Ridge distorted-wave codes[3,4,21] conduct a search which adjusts the potential parameters until the mean-square deviation of the prediction for the elastic cross section from the observed cross section is a minimum. The optical-model potential is assumed to have a Woods-Saxon shape,

$$U(r) = -(V_0 + iW_0)(e^x + 1)^{-1} \qquad (1)$$

where $x = (r - R_0)/a$. Figure 8 shows the experimental elastic-scattering differential cross section. The DWBA prediction (solid curve) was calculated with $R_0 = 5.64$ F, $a = 0.651$ F, $V_0 = 30.57$ MeV, and $W_0 = 17.18$ MeV.

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[19] E. Newman, R. H. Bassel, R. S. Bender, J. R. Donaldson, and K. S. Toth, Bull. Am. Phys. Soc. **9**, 57 (1964).

[20] E. A. Silverstein, Nucl. Instr. Methods **4**, 53 (1959).

[21] R. H. Bassel, G. R. Satchler, and R. M. Drisko, *Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, 1963*, edited by A. Ghiorso, R. M. Diamond, and A. E. Conzett (University of California Press, Berkeley, 1963), p. 45.

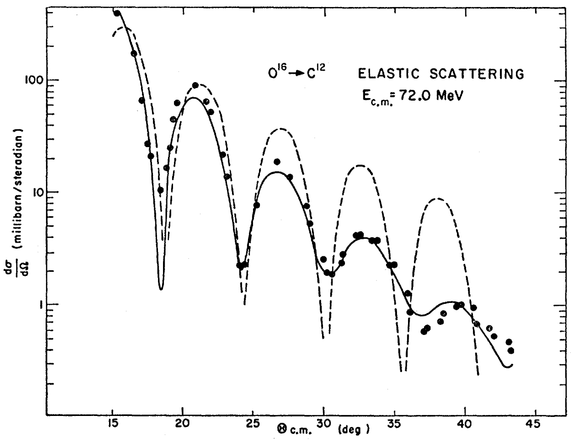

FIG. 8. Comparison of the measured elastic differential cross section to the DWBA prediction (solid curve) and to the plane-wave Born approximation prediction (dashed curve).

The inelastic-scattering transition amplitude is given by

$$T_{fi}=\int dr\chi_f^{(-)*}(\mathbf{k}_f,\mathbf{r})\langle v_f|V|v_i\rangle\chi_i^{(+)}(\mathbf{k}_i,\mathbf{r}),\qquad(2)$$

where v_i and v_f refer to the initial and final nuclear states, V is the interaction causing the inelastic transition, and the χ's are the distorted waves describing the elastic scattering of the projectile before and after the inelastic transition. In the absence of a Coulomb field they have the asymptotic forms

$$\begin{aligned}\chi^{(+)}(\mathbf{k},\mathbf{r})&=\exp(i\mathbf{k}\cdot\mathbf{r})+f(\Theta)\exp(ikr)/r,\\ \chi^{(-)}(\mathbf{k},\mathbf{r})&=\exp(i\mathbf{k}\cdot\mathbf{r})+f(\pi-\Theta)\exp(-ikr)/r.\end{aligned}\qquad(3)$$

These distorted waves satisfy the Schrödinger equation

$$[\nabla^2+k^2-(2\mu/\hbar^2)\{U(r)+U_c(r)\}]\chi(\mathbf{k},\mathbf{r})=0,\qquad(4)$$

where U_c is the Coulomb potential. It is important to note that the DWBA assumes that the elastic scattering is the dominant process and treats the potential inducing the inelastic scattering as a perturbation. The strength of the inelastic reactions in this system (see Fig. 7) indicates that this approximation may not be a good one. Buck[22] has considered an extension of the optical model in which the quadrupole collective state is strongly coupled to the nuclear ground state. His work shows that for β_2 greater than about 0.2, it is necessary to use the coupled-equations formalism rather than the DWBA. In the (p,p') cases he considers, the DWBA underestimates the values of β_2. For example, when the DWBA produces a $\beta_2=0.30$, the coupled equations yield $\beta_2=0.35$. However, Perey and Satchler[23] have recently pointed out that these results are misleading, and that if the DWBA is applied consistently, this approximation has a wider range of validity. Further work by the Oak Ridge group has shown that a consistent application of the DWBA gives reliable results for the inelastic scattering of heavy ions.[24]

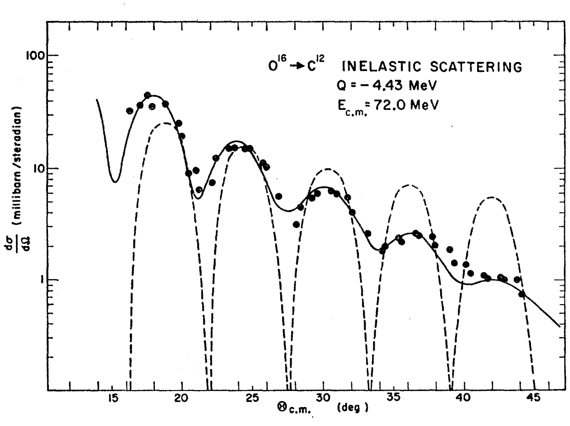

FIG. 9. Comparison of the measured inelastic differential cross section for the excitation of the 4.43-MeV level in C^{12} to the DWBA prediction (solid curve) and to the plane-wave Born approximation prediction (dashed curve).

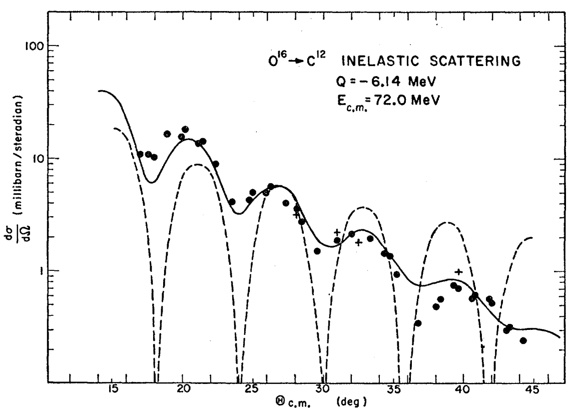

FIG. 10. Comparison of the measured inelastic differential cross section for the excitation of the 6.14-MeV level in O^{16} to the DWBA prediction (solid curve) and to the plane-wave Born approximation prediction (dashed curve). Data points shown as crosses indicate kinematic coincidence techniques used.

The DWBA fits to the inelastic-scattering angular distributions shown as solid curves in Figs. 9–11 were normalized to fit the maxima of the cross sections. The values of β_L obtained from these fits are indicated in Table I.

Figure 10 shows the measured angular distribution for the 3^- state in O^{16} at 6.14 MeV. It is noted again that the energy resolution in this experiment is not sufficient to separate from this cross section any contributions from the excitation of the 0^+ state in O^{16} at 6.06 MeV. Experimental evidence in the C^{12}–C^{12} system[9,18] and $C^{12}(\alpha,\alpha')C^{12*}$ reactions[25] shows the excitation of the 0^+ state in C^{12} at 7.66 MeV to be strongly inhibited.

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[23] F. Perey and G. R. Satchler, Phys. Letters **5**, 212 (1963).
[24] R. H. Bassel, R. M. Drisko, and G. R. Satchler (to be published).
[25] University of Washington, Cyclotron Progress Report, 1962 (unpublished).

Excitation of a spin-0 state is inhibited relative to other states by a $2L+1$ statistical factor and further, may have a wave function very different from the ground state. Hence it is expected that the 0^+ state in O^{16} should also be only weakly excited and the agreement between the experimental angular distribution and the negative parity fit shown in Fig. 10 bears out this contention. Since a static octupole deformation is parity forbidden, it is preferable to use the vibrational-model parametrization in this instance setting

$$\beta_L{}^2=(2L+1)(\hbar\omega_L/2C_L), \quad (5)$$

where C_L is related to the surface tension of the deformed nucleus. The resulting value of C_3 is 380 MeV.

The $Q=-7.0\pm0.1$-MeV cross section is shown in Fig. 11. The uncertainties in these data are large because of poor statistics and background problems. Thus the phase of the oscillation is difficult to determine. The positive parity fit shown (dashed curve) is not very convincing but when the phase of the oscillations in this cross section is compared with the oscillations in the $Q=-6.14$-MeV cross section (see Fig. 7), a positive parity assignment to the 7-MeV excitation seems reasonable. Thus, this cross section is attributed primarily to the excitation of the 2^+ state at 6.92 MeV in O^{16} although there are probably contributions from the 1^- state at 7.12 MeV. These contributions fill the minima of the positive-parity cross section. Thus the value of β_2 quoted for this state represents an upper limit.

TABLE I. Inelastic-scattering transition strengths.

Nucleus	$-Q$(MeV)	L	β_L(DW)	$\beta_L R_0$(DW)
C^{12}	4.43	2	0.30	1.69 F
O^{16}	6.14	3	0.24	1.35 F
O^{16}	6.92	2	$\leq$0.16	$\leq$0.90 F

The DWBA predictions include Coulomb excitation effects which result in a small renormalization of the $Q=-4.43$-MeV excitation and has little effect on the other excitations.[24] The DWBA transition strength for the excitation of the 4.43-MeV state in C^{12} is found to be slightly smaller than the value 0.36 determined from the C^{12}–C^{12} system[21] or the value 0.38 from (α,α') experiments.[26]

The DWBA deformation parameters in Table I have been extracted from the data using the radius parameter $R_0=R_0{}^{(1)}+R_0{}^{(2)}$. Recently, a convention has been adopted wherein the procedure is to use only the radius parameter of the nucleus being excited.[21,24] With this convention the quoted values of β_L (DWBA) in Table I should be approximately doubled. Blair has suggested[27]

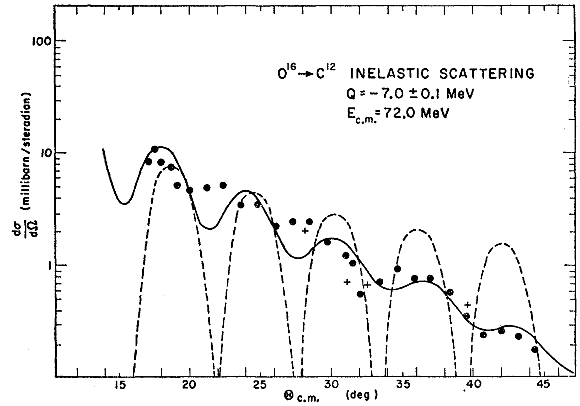

FIG. 11. Comparison of the measured inelastic differential cross section for the 7.0 ± 0.1-MeV excitation in O^{16} with the DWBA prediction for the quadrupole state at 6.92 MeV (solid curve) and the plane-wave Born approximation prediction for the same state (dashed curve). Data points shown as crosses indicate kinematic coincidence techniques used.

that $\beta_L R_0$ rather than β_L should be extracted from the data since it is the former quantity that directly enters the expansion of the potential in this theory. This quantity is independent of the question of the proper radius parameter and has been shown to be relatively independent of the projectile used in exciting a state in a target nucleus.[21,24] The experimental values of $\beta_L R_0$ determined by the DWBA fit to the data are included in Table I.

(b) Mutual Excitation Reaction

The last angular distribution measured is for the mutual excitation reaction in which the 4.43-MeV state in C^{12} and the 6.14-MeV state in O^{16} are both excited. The experimental data and a plane-wave Born approximation (PWBA) prediction are shown in Fig. 12. The

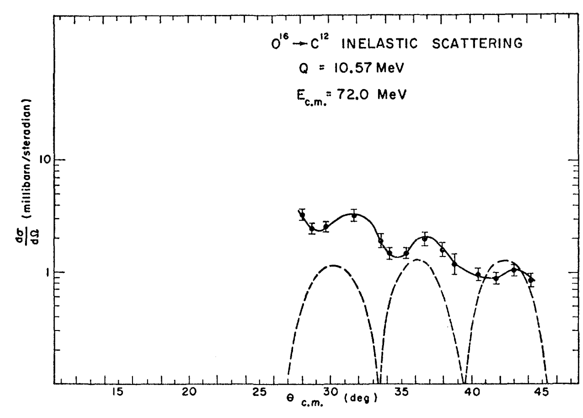

FIG. 12. Comparison of the measured differential cross section for the mutual excitation of the 4.43-MeV state in C^{12} and the 6.14-MeV state in O^{16} (solid curve) to the plane wave Born approximation prediction (dashed curve).

[26] E. Rost, doctoral dissertation, University of Pittsburgh, 1961 (unpublished).

[27] J. S. Blair, *Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962*, edited by E. Clementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963), p. 669.

data could not be extended to more forward angles because the kinetic energy of the recoil nucleus becomes small and the coincidence efficiency is decreased because of multiple scattering in the target.

Unfortunately, a DWBA calculation of this reaction is not possible. Although the PWBA theory of second order processes has been shown to be incorrect[28,29] it gives a good agreement with experiment in the C^{12}–C^{12} case[9] and again in the present case. A correct theory of second-order processes within the context of the adiabatic-Fraunhofer method has been developed[30] and the application of this theory to the present case is to be published shortly.[24]

The application of the PWBA theory to heavy ion reactions has been discussed in detail in Ref. 9. The predictions of this theory are shown as dashed curves in Figs. 8–12. The parameters used[31] in obtaining the PWBA fits are $V_0=4.46$ MeV, $R_0=6.85$ F, $\beta_2(C^{12})=0.14\pm0.02$, $\beta_3(O^{16})=0.09\pm0.02$, and $\beta_2(O^{16})\leqq0.07$. These parameters are determined by the elastic cross section and the cross sections for the single excitation of the states involved. No free parameters remain for fitting the mutual-excitation cross section. As seen in Fig. 12 the agreement between experiment and theory is quite good, both with respect to phase and magnitude.

IV. DISCUSSION

(a) Comparison with Other Reactions

A useful technique for comparing the present experimental results with previous experiments utilizing different projectiles is inherent in the adiabatic diffraction theory.[1] In this theory the elastic-scattering cross section has the form

$$d\sigma/d\Omega=[k_0R_0^2]^2[J_1(k_0R_0\Theta)/k_0R_0\Theta]^2, \qquad (6)$$

where $J_1(k_0R_0\Theta)$ is the cylindrical Bessel function of order one. Note that the normalization depends only on the incident momentum and the nuclear size and is independent of the details of the interaction. Thus if the assumptions of the adiabatic diffraction model are met the cross sections of different projectiles corrected for momentum and size should yield a universal curve[32,33] when plotted against $k_0R_0\Theta$.

A plot of this nature is shown in Fig. 13. The reactions shown include the scattering of protons, alpha particles, and heavy ions from C^{12}. It is noted that the heavy ion results agree quite well. The C^{12}–O^{16} data do not show a strong minimum at $k_0R_0\Theta=7$, possibly because of the lack of angular resolution in the experiment.[12] The alpha-particle results compare well with the heavy-ion results for $k_0R_0\Theta\lesssim9$ but deviate quite sharply at higher values of momentum transfer. The assumptions of the adiabatic diffraction analysis are not applicable to large-angle alpha-particle scattering and further, preformation of alpha particles[34] in C^{12} could account for deviations from simple diffraction scattering. Recent studies[35] of the scattering of alpha particles from C^{12} indicate anomalous, energy-dependent behavior at backward angles.

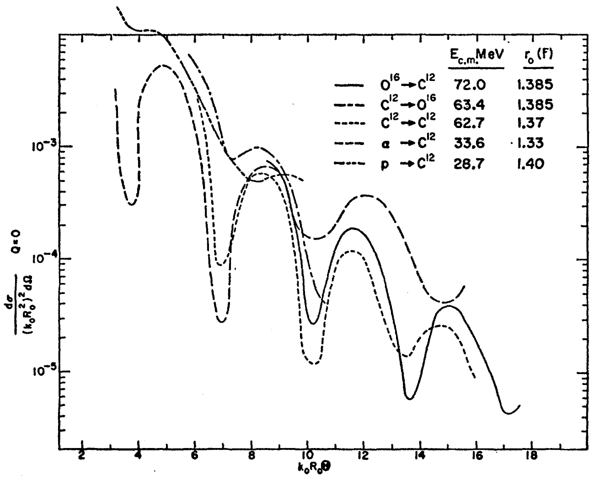

FIG. 13. Comparison of the elastic scattering of different projectiles from C^{12} within the framework of the adiabatic diffraction scattering model. The data used where obtained from Refs. 9, 10, 25, 36, and the present work.

The proton scattering is seen to agree roughly with the heavy-ion data but the diffraction oscillations are not as strong in the latter case. This is understandable in that protons of this energy have a longer mean free path in nuclear matter so the black disk model for the nucleus is not correct.[36] Thus, it is expected that interference effects resulting from scattering within the nuclear volume as well as at the nuclear surface will reduce the peak to valley ratio.

A similar comparison is shown for the inelastic scattering to the 2^+ state in C^{12} at 4.43 MeV, in Fig. 14. In this case the heavy-ion results again agree very well but both the alpha particle and low-energy proton-scattering cases are apparently enhanced for all values of momentum transfer. Further studies of these reactions are necessary to gain an understanding of the exact nature of these differences.

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[28] B. Buck, Phys. Rev. **127**, 940 (1962); N. Austern, R. M. Drisko, E. Rost, and G. R. Satchler, Phys. Rev. **128**, 733 (1962).

[29] N. S. Wall, *Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962*, edited by E. Clementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963), p. 208.

[30] J. S. Blair and N. Austern (unpublished).

[31] For a complete discussion of the PWBA calculation and definition of the parameters involved see Ref. 9.

[32] J. S. Blair, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960), p. 824.

[33] J. S. Blair, D. Sharp, and L. Wilets, Phys. Rev. **125**, 1625 (1962).

[34] G. Igo, Phys. Rev. Letters **7**, 29 (1961).

[35] R. A. Atneosen, H. L. Wilson, M. G. Sampson, and D. W. Miller, Bull. Am. Phys. Soc. **8**, 303 (1963).

[36] J. K. Dickens, D. A. Haner, and C. N. Waddell, Phys. Rev. **129**, 743 (1963).

It is of interest to note that the inelastic scattering of protons, deuterons, alpha particles, and heavy ions strongly excite the same levels in target nuclei, i.e., the collective 2^+ and 3^- levels. The excitation of the 0^+ states in C^{12} and O^{16} is found to be reduced by about a factor of 10 from the strongest excitations. This is true for protons up to energies of 150 MeV.[36,37] In fact, it is only in the (e,e') scattering that the excitation of 0^+ states is strong. The (e,e') data for C^{12} seem to be explainable by considering the 0^+ state to be a collective state formed by coupling two quadrupole phonons to spin 0.[38]

(b) Inelastic-Scattering and Electromagnetic-Transition Probabilities

Comparison of the DWBA theory of inelastic scattering to experiment yields a normalization constant $\beta_L R_0$ which contains the familiar nuclear-deformation parameter. The same parameter, β_L, to be called a transition strength hereafter, also appears in the theory of the excitation of collective levels through electromagnetic interactions,[39] i.e., by Coulomb excitation, (e,e') reactions or radiative decay. Thus it is possible to compare values of transition strengths obtained from inelastic scattering of various projectiles and electromagnetic transitions.

The relationship between electromagnetic interactions and inelastic scattering can be utilized to compare the cross sections for inelastic scattering from collective states in different nuclei. The cross section for inelastic scattering from a given state in a nucleus is related to the strength of the radiative transition to the ground state

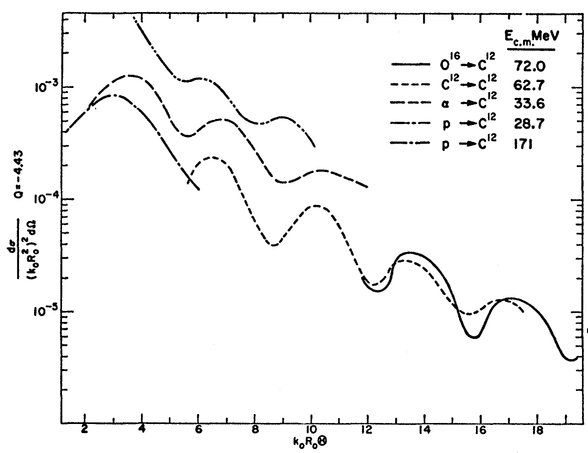

FIG. 14. Comparison of the inelastic scattering to the first excited state in C^{12} within the framework of the adiabatic diffraction scattering model. The data used were obtained from Refs. 9, 25, 36, 37, and the present work.

TABLE II. Inelastic versus electromagnetic transition strengths.

Nucleus	$-Q$(MeV)	L	$\Gamma_\gamma/\Gamma_{\gamma W}$	$\beta_L R_0$(DW)	$\beta_L R_0$(EM)
C^{12}	4.43	2	3.2	1.69 ± 0.25 F	1.49 ± 0.16 F
O^{16}	6.14	3	13.6	1.35 ± 0.20 F	2.47 ± 0.7 F
O^{16}	6.92	2	1.1	$\leqq 0.90\pm0.13$ F	0.68 ± 0.09 F

through the reduced transition probability $B(\mathrm{E}L)$ associated with the electric-multipole transition connecting these states. The reduced transition probabilities can be obtained from experimental data on electromagnetic transitions and then used to predict inelastic-scattering cross sections. In making this prediction a nuclear model must be invoked to relate the inelastic-scattering cross section to the reduced transition probability. This relationship will be presented below using the collective model of Bohr and Mottelson.[40] It may thus be possible to test the collective model by comparing the cross section for the excitation of collective states by inelastic scattering with the predictions obtained from electromagnetic interactions.

Comparing Inelastic and Electromagnetic Transition Strengths

The radiative transition probability is related to the reduced transition probability through the well-known relation[39]

$$T=\frac{8\pi(L+1)}{[(2L+1)!!]^2}\frac{1}{\hbar}\left[\frac{\Delta E}{\hbar c}\right]^{2L+1}B(\mathrm{E}L). \qquad (7)$$

The reduced transition probability is model dependent and is given in the collective model,[40] by

$$B(\mathrm{E}L; L\to 0)=(9/16\pi^2)Z^2e^2R_e^{2L}\beta_L^2/(2L+1), \qquad (8)$$

assuming a uniform, spheroidal charge distribution of $R_e=r_eA^{1/3}$ with r_e given by electron scattering experiments. In these equations ΔE is the energy of the transition and $1/T$ is the lifetime of the excited state. Table II compares the quantity $\beta_L R_0$ obtained from the DWBA analysis of this data and the C^{12}–C^{12} data[24] with the same quantity obtained by applying Eqs. (7) and (8) for $E2$ transitions to existing experimental data on lifetimes[16] and (e,e') cross sections.[38] In these calculations the radius parameter is assumed to be $r_e=1.36$ F as given by the electron-scattering experiments. The electromagnetic enhancements, $\Gamma_\gamma/\Gamma_{\gamma W}$ where Γ_γ is the observed radiative width of the transition and $\Gamma_{\gamma W}$ is the Weisskopf single-particle estimate, are also included in Table II. Again, the β_2(DW) for O^{16} may be overestimated by as much as a factor of 1.5 since the experiment includes the possible excitation of the 1^- state at 7.12 MeV. It is interesting to note that the value of $\beta_3 R_0$ from this experiment is in good agreement

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[40] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).

with the value of 1.57 obtained from a DWBA fit of (p,p') data.[41]

The inelastic-scattering transition strengths are generally smaller than the corresponding electromagnetic values. This tendency has also been noted in (p,p') and (α,α') reactions.[4,22] The discrepancy is worse in the case of octupole transitions. This may arise from the failure to consider the angular momentum and energy dependence of the nuclear matrix elements for inelastic scattering.

Relating Cross Sections in Different Nuclei

Thus far, inelastic-scattering reactions have been seen to strongly excite the collective 2^+ and 3^- states in nuclei, indpendent of the projectile used in the reactions. A method of comparing these cross sections is proposed by relating the normalization for the cross section[2] which depends on the transition strength and mass number as

$$d\sigma_L/d\Omega \propto k^2\beta_L{}^2r_0{}^4A^{4/3}\,, \tag{9}$$

and the reduced transition probability, in the collective model from Eq. (8)

$$B(\mathrm{EL}) \propto Z^2\beta_L{}^2r_e{}^{2L}A^{2L/3}\,. \tag{10}$$

A distinction is made between the reduced electromagnetic radius r_e which is smaller than the reduced nuclear radius r_0 determined from inelastic scattering experiments. However, it is assumed that the nuclear charge distribution has the same shape as the nuclear mass distribution and thus Eqs. (9) and (10) can be combined to give, in the limit of a collective model,

$$(d\sigma_L/d\Omega) \propto (r_0{}^4/r_e{}^{2L})(k^2B(\mathrm{EL})/Z^2)A^{(2/3)(2-L)}\,. \tag{11}$$

Thus, even though a state may show strong electromagnetic enhancement in a heavy nucleus, the charge dependence indicated in Eq. (11) may result in the observation of a small inelastic cross section. Equation (11) is based on the crude assumption of a uniformly charged spheroidal nucleus and might only be expected to explain the systematic variation of cross sections with atomic number over the whole periodic table. It obviously cannot explain variations in cross section between isotopes. However, this variation of cross section with atomic number is observed in the present experiment. Using Eq. (11), the predicted ratio of the 2^+ cross sections in O^{16} and C^{12} is found to be

$$\frac{(d\sigma_2/d\Omega)(\mathrm{O}^{16})}{(d\sigma_2/d\Omega)(\mathrm{C}^{12})}=\frac{B(E2)\mathrm{O}^{16}}{B(E2)\mathrm{C}^{12}}\frac{Z_c{}^2}{Z_0}=0.27\pm0.07\,. \tag{12}$$

This experiment yields a maximum value for this ratio of 0.21 ± 0.05. Using 150-MeV protons Rowe *et al.*[37] found this ratio to be 0.24 ± 0.04. It is interesting to note that the relative strengths of the inelastic scattering from these states are very nearly the same for proton and heavy ion interactions. This would indicate that the features of (x,x') reactions are not strongly dependent on projectile type when collective states in the target (or projectile) are excited.

Comparisons such as the one above are useful in testing the charge dependence of the reduced transition probability. Equation (8) is a first approximation to this relationship and is tested in Table III with some available (α,α') data on quadrupole excitations. Table III compares three ratios. The ratio of the differential cross section for exciting the first 2^+ state in a given nucleus to the cross section for exciting the 4.43-MeV state in C^{12} with 42-MeV alpha particles[25] is shown in the last column. The differential cross sections are taken at the same peak in the angular distributions, located at c.m. angle 37° for C^{12} and near c.m. angle 20° for the heavier nuclei. These cross-section ratios are only accurate to approximately ±30%. Column 4 contains the ratio of the reduced transition probabilities for the same states, calculated from observed lifetimes[16] using Eq. (7) or obtained from Coulomb excitation measurements.[42] The fifth column contains the predicted ratio for the cross sections using Eq. (11). These calculations assume that the ratio of the radius parameters, r_0/r_e, remains constant over the range of nuclei considered. The table shows that the ratios calculated using Eq. (11) are clearly in better agreement with the ratios of the observed cross sections than are the simple ratios of the reduced transition probabilities. A similar result is found in the present experiment where the ratio of the reduced transition probabilities for the states considered in Eq. (12) is 0.52 and the weighted ratio is in better agreement with the observed cross sections.

Although the weighted ratios are in better agreement with the observed cross sections, it is apparent that a more accurate calculation of the reduced transition probabilities than Eq. (8) is necessary to achieve even

TABLE III. Relationship of (α,α') cross sections to reduced transition probabilities for quadrupole transitions.

Nucleus	E_i (MeV)	$-Q$ (MeV)	$B(E2)/B(E2)_{\mathrm{C}^{12}}$	$(B/B_c)\times[6/z]^2$	$d\sigma/d\sigma_{\mathrm{C}^{12}}$
Mg^{24}	42	1.37	11.0	2.76	1.41[a]
A^{40}	43[b]	1.46	4.9	0.54[a]	0.77
Ti^{48}	43	0.99	22.1	1.63	0.71[c]
Ni^{58}	43	1.33	28.8	1.33	1.41[a]
Zn^{66}	43	1.04	58	2.32	1.82[d]

[a] Experimental cross sections and $B(E2)$ for A^{40} taken from Ref. 4.
[b] Experimental cross section scaled up to $E_i=43$ MeV using Eq. (9) from the 18 MeV data of L. Seidlitz, E. Bleuler, and D. J. Tendam, Phys. Rev. **110**, 682 (1958).
[c] Experimental cross section taken from H. W. Broek, *Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms Padua, 1962*, edited by E. Clementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963), p. 770.
[d] Experimental cross section taken from Ref. 44.

[41] G. R. Satchler, R. H. Bassel, and R. M. Drisko, Phys. Letters **5**, 256 (1963).

[42] P. H. Stelson and F. K. McGowan Nucl. Phys. **32**, 652 (1963).

reasonable agreement with the cross-section ratios. Thus, the description of nuclei showing strong collective oscillations as deformed, uniformly charged spheroids is not adequate and it would seem that the inclusion of shell effects is necessary. A similar consideration of (p,p') scattering[43] shows the cross section for exciting collective quadrupole states to have a smoothed-out dependence on A characterized by A^{-n} with $n\simeq 0$ to 2 and further, indicates the possibility of a shell effect. Another indication of the need for a more accurate calculation of the charge dependence of $B(E2)$ has been noted by Broek[44] who found a significant difference in the ratio σ/β^2 between $Z=28$ and $Z=30$ using (α,α') reactions.

V. CONCLUSION

The most strongly excited states in the scattering of O^{16} from C^{12} are those that show enhancement of their electromagnetic transition probabilities to the ground states, i.e., the 2^+ state in C^{12} at 4.43 MeV and the 3^- state in O^{16} at 6.14 MeV. However, since O^{16} was used as the beam and was the nucleus detected in the forward counter, particle-unstable collective states excited in this nucleus could not be observed. Thus, the fact that a strong 2^+ excitation near 18 MeV was not observed in no way argues against the existence of such a level. This collective 2^+ level predicted by Fallieros and Ferrell,[45] would most likely be particle unstable and would have to be detected in the inverse experiment, C^{12} on O^{16}.

The excitation of the 3^- state in O^{16} is understood in terms of an octupole vibration which Brown and his coworkers[46] have shown in the case of doubly-magic nuclei to result from the particle-hole interaction. The values obtained for the transition strength, both from inelastic proton scattering and this work agree well with one another but are at least a factor of 2 smaller than the value obtained from the electromagnetic lifetime. In a crude collective model the inelastic transition strengths should be exactly the electromagnetic value. However, in more refined models it is quite possible that appreciable differences may occur in that the electromagnetic operator is summed over proton states only, whereas for inelastic scattering the sum would have to include neutron states also.

Brown and Thouless[46] also note that a dipole, $T=0$ collective state cannot exist since it is a spurious vibration of the center of mass. Thus, the results of the Oak Ridge experiment[19] are needed to resolve the question of which states contribute to the 7-MeV excitation in the O^{16}–C^{12} reaction.

The apparent lack of excitation of the low-lying excited states of spin 0 in both C^{12} and O^{16} is puzzling. The C^{12} case has been discussed in Ref. 9. In O^{16} the $E2$ matrix elements for the 2^+ (6.92 (MeV) to 0^+ (6.06 MeV) and 2^+ (6.92 MeV) to 0^+ (g.s.) are comparable and quite strong.[47] Thus the 2^+ and 0^+ excited states must be closely related and the 0^+ should be excited, although statistical factors will reduce the yield. Again, experiments with better energy resolution are necessary to allow the separation of the 0^+ and 3^- excitations.

A plane-wave Born approximation has been used to obtain a qualitative fit to the data. The primary value of this approximation is in providing a simple technique for calculating the mutual-excitation cross section. The prediction of the plane-wave approximation is remarkably good considering that all free parameters in the theory are fixed by the elastic scattering and the first-order excitations. Detailed calculations within the DWBA framework[28] indicate that 2nd Born approximation contributions are significant for 40-MeV (α,α') reactions but for the heavy-ion cases the direct term is larger than the successive term and the phase predicted by the plane-wave approximation should be nearly correct.[24]

From the consideration of the relationship between inelastic-scattering data and electromagnetic transition probabilities in Table III it would appear that this relationship can be used to gain a better understanding of collective wave functions in nuclei. Equation (11) rests on the assumption of a uniform, spheroidal nuclear-charge distribution in calculating the reduced transition probabilities. A systematic study of inelastic scattering from quadrupole and octupole states may yield more precise information on how the nuclear charge enters into radiative transitions.

ACKNOWLEDGMENTS

The authors wish to express appreciation to Professor E. R. Beringer and Professor D. A. Bromley for their interest and suggestions concerning this work. We are deeply indebted to Dr. R. H. Bassel of Oak Ridge National Laboratory for providing the DWBA fits to the experimental data and for several helpful discussions concerning the interpretation of the theory. Finally, the efforts of the heavy-ion accelerator staff to provide the requested beam and services are gratefully ackowledged.

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